Common Final Exam

Calculus I, Math 161, Spring 2024

Name: _____

Question	Points	Score
1	25	
2	10	
3	25	
4	10	
5	20	
6	10	
7	15	
8	10	
9	10	
10	10	
11	10	
12	20	
Total:	175	

- No books or notes of any kind are allowed
- No technology – calculators, cell phones, or computers – is allowed
- Show your work!
- You have 120 minutes to complete this exam.

1. Compute the following limits:

   (a) (5 points) $\lim_{x \to 4} \dfrac{x-4}{x^2-2x-8}$

   (b) (5 points) $\lim_{t \to \infty} \dfrac{1-15t^2}{3t^2-t+2}$

(c) (5 points) $\lim_{x \to \pi^+} \dfrac{2\cos(3x) + 5x^2 + 3}{3x + \sin(x/2)}$

(d) (5 points) $\lim_{x \to 0}(\sqrt{x^2 + 1} - x)$

(e) (5 points) $\lim_{x \to 0} \dfrac{x^2}{1 - \cos(2x)}$

2. (10 points) Find the vertical and horizontal asymptotes of $f(x) = \dfrac{2x^2 + x - 1}{x^2 + x - 2}$

3. Find the derivatives of the following functions.

   (a) (5 points) $y = (2x^3 - 5x^2 + 9x)^3$

   (b) (5 points) $y = \dfrac{x^2 - x + 2}{\sqrt{x}}$

(c) (5 points) $y = 5^{\tan \pi\theta}$

(d) (5 points) $f(w) = \cos^2\left(\sqrt{\sin(\pi w)}\right)$

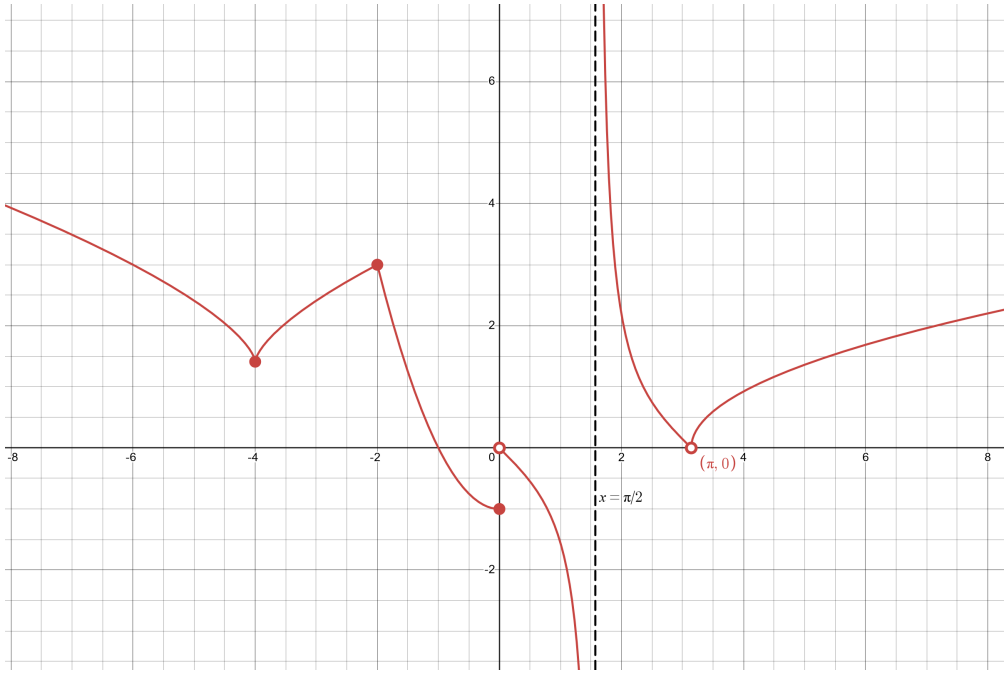
(e) (5 points) Use implicit differentiation to find $\dfrac{dy}{dx}$ if $x^2 + xy + y^2 + 1 = 0$.

4. (10 points) Find the equation of the tangent line to the curve

$$x(t) = t^2 + 5t + 4 \qquad y(t) = 4t$$

at the point $(4, 0)$.

5. Here is a graph of a function $g(x)$



(a) Use the graph to compute the following limits, or state that they do not exist:

i. (2 points) $\lim_{x \to -2} g(x)$

ii. (2 points) $\lim_{x \to 6} g(x)$

iii. (2 points) $\lim_{x \to \frac{\pi}{2}^-} g(x)$

iv. (2 points) $\lim_{x \to 0} g(x)$

v. (2 points) $\lim_{x \to 0^+} g(x)$

(b) (5 points) For what values of x is $g(x)$ discontinuous?

(c) (5 points) For what values of x is $g(x)$ not differentiable?

6. (10 points) Use a linear approximation to estimate tan(3), showing all work. Give your answer to two decimal places.

7. (15 points) Find all critical numbers, all intervals where the function is increasing or decreasing, all inflection points, and all intervals where the functions is concave up or concave down, for $f(x) = x^4 - 2x^2 - 8$.

8. (10 points) Find the absolute max and min value of $F(x) = x^2 - 8\ln(x)$ on $[1, 10]$. Show all your work. You may use the approximate values for $\ln(x)$ in the following table.

x	1	2	3	4	5	6	7	8	9	10
$\ln(x)$	0	0.69	1.1	1.39	1.61	1.79	1.95	2.08	2.20	2.30

9. (10 points) The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

10. (10 points) Approximate the integral

$$\int_{-3}^{-1} 4^{-x}\,dx$$

using a left endpoint Riemann sum with four equal sub-intervals.

11. (10 points) Find

$$\frac{d}{dx}\int_{\sqrt{3}}^{x} \frac{t^3+t}{\sqrt{\sin(t)}}\,dt$$

(Hint: Don't compute the integral directly.)

12. Evaluate the following integrals:

(a) (5 points) $\int \left(\frac{t+1}{\sqrt{t}} + \sec^2(t) \right) \, dt$

(b) (5 points) $\int_1^3 x^2 - \frac{1}{x} + 6e^x \, dx$

(c) (5 points) $\int t \sin(5t^2)\, dt$

(d) (5 points) $\int_1^2 x\sqrt{x-1}\, dx$